A six-rectangle Venn diagram

GPT-5.6 Pro
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A six-set Venn diagram consists of six simple closed curves such that every choice of interiors and exteriors is nonempty and connected. Ruskey and Weston ask whether the curves can all be rectangles and attribute the problem to Grünbaum [RW05]. We give a computer-assisted construction with rational coordinates and an exact certificate. For $\varepsilon \in \{0,1\}^6$, let B_ε be the set of points off the boundaries that lie in the ith rectangle R_i exactly when $\varepsilon_i = 1$; we prove that every B_ε is one face. An interactive Desmos version is available from `https://www.desmos.com/calculator/bf94x051xc`.

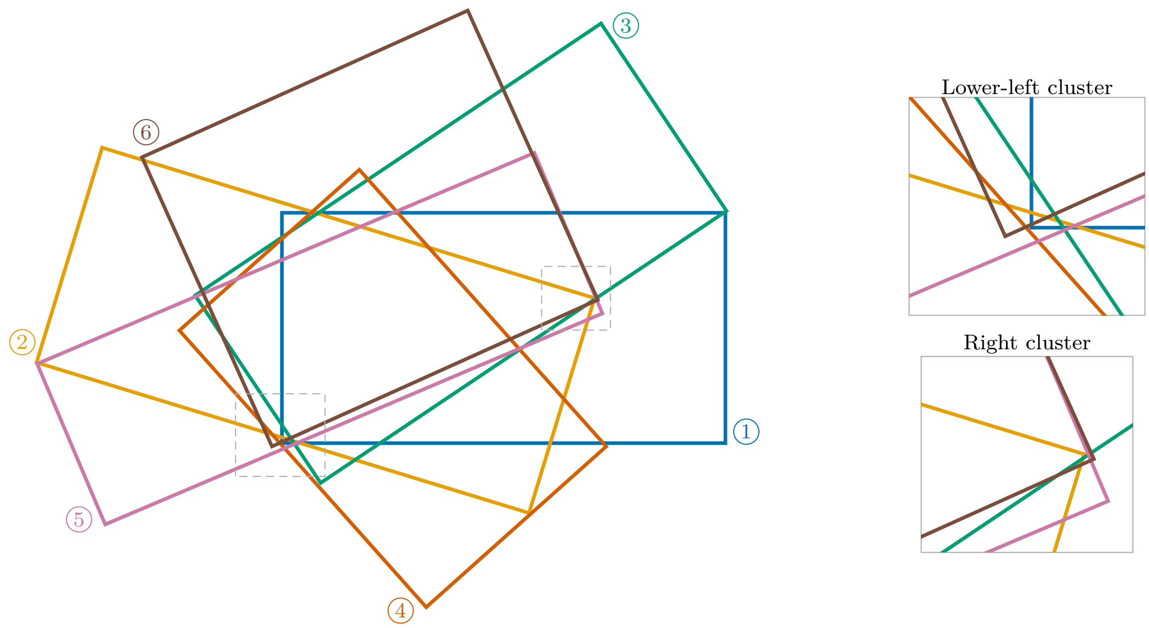

Figure 1. The six rectangular curves. The dashed boxes are enlarged on the right.

Exact construction. Set $D = 10^4$, and put

$$c_i = (c_{i,x}, c_{i,y}), \qquad u_i = (p_i, q_i), \qquad v_i = \lambda_i(-q_i, p_i), \qquad R_i = \{c_i + su_i + tv_i : |s|, |t| \le 1\},$$

following the values given in the table below. Thus the vertices are $c_i - u_i - v_i$, $c_i + u_i - v_i$, $c_i + u_i + v_i$, and $c_i - u_i + v_i$. Since $u_i \cdot v_i = 0$ and $\lambda_i > 0$, each R_i is a nondegenerate rectangle.

i	$Dc_{i,x}$	$Dc_{i,y}$	Dp_i	Dq_i	$D\lambda_i$
1	3196	−488	6447	0	5191
2	−2273	−556	7159	−2186	4359
3	1950	1685	5900	3951	4616
4	−28	−2245	3588	−4020	6507
5	−2151	−798	7228	3065	3227
6	−694	2402	1890	−4201	11274

Theorem. *For the rectangles in the table, every B_ε is a single face. Hence $\partial R_1, \ldots, \partial R_6$ form a six-set Venn diagram.*

Proof (exact computation). The accompanying Python verifier, available at `https://boonsuan.github.io/venn.py`, uses exact rational arithmetic via `fractions.Fraction`. It finds 62 distinct proper crossings, with no tangency, triple crossing, or corner on another boundary. The pairwise crossing numbers are

$$(n_{ij}) = \begin{pmatrix} 0 & 4 & 6 & 4 & 4 & 2 \\ 4 & 0 & 4 & 4 & 4 & 4 \\ 6 & 4 & 0 & 4 & 4 & 6 \\ 4 & 4 & 4 & 0 & 4 & 4 \\ 4 & 4 & 4 & 4 & 0 & 4 \\ 2 & 4 & 6 & 4 & 4 & 0 \end{pmatrix}, \qquad \sum_{i<j} n_{ij} = 62.$$

Every pair of boundaries meets, so their arrangement graph is connected. Its vertices are the 24 rectangle corners and the 62 crossings. Each crossing splits two sides, and therefore

$$V = 24 + 62 = 86, \qquad E = 24 + 2 \cdot 62 = 148, \qquad F = E - V + 2 = 64.$$

The equality $F = 64$ does not by itself identify the Boolean regions. The verifier therefore sorts the crossings along each side, reconstructs the cyclic order at every vertex, and traverses all faces. It labels the unbounded face by 000000 and toggles bit i whenever a dual edge crosses ∂R_i. The labels obtained are exactly the 64 words in $\{0,1\}^6$, each once. Thus every B_ε is nonempty and connected. The minimum bounded-face area is positive and is approximately 9.4555×10^{-6}, so no face is produced by a zero-area degeneracy. This proves the theorem. ☐

How it was found. A six-triangle Venn arrangement of Carroll [Car00] was used as a seed. One unused outer corner of each triangle was truncated, and numerical continuation moved the supporting lines to two pairs of parallel, perpendicular sides while preserving the crossing order and face labels. A final local optimization enlarged the smallest faces. After rounding all parameters to denominator 10^4, the exact verification was repeated.

References. [Car00] J. J. Carroll, *Drawing Venn Triangles*, HPL-2000-73 (2000). [Gr84b] Branko Grünbaum, *On Venn Diagrams and the Counting of Regions*, The College Mathematics Journal, vol. 15, no. 5 (1984), 433–435, doi:10.2307/2686559. [RW05] F. Ruskey and M. Weston, *Venn Diagrams*, Electron. J. Combin. Dynamic Survey DS5 (2005), doi:10.37236/26; open-problem list.

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